Abstract

The mechanical characterization of mortars is problematic due to their sensitivity to stress concentration defects, namely pores and flaws. In this paper, data on three-point bending, Young’s modulus and fracture toughness of mortar beams are presented and discussed. Here, Weibull statistics is applied to analyse flexural strength of the developed mortar. Based on data obtained according to the EN196-1 standard and corrected values taking into account the actual position where failure origin took place, Weibull modulus was found to decrease from 28 to 22, respectively. In addition, fracture toughness was determined using Griffith approach based on critical crack sizes measured by fractography in fracture surfaces. A value of $0.37\pm0.04$ MPa m$^{\frac{1}{2}}$ was obtained, which is typical of very fragile brittle materials.

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1. Introduction

Nowadays, mortars are widely used for both new construction and repair. Strengths of mortars change based on the ratios of water/cement and sand/cement which are used. Being a very brittle material, it is crucial to establish an approach that can be incorporated into the design of reliable components. A common technique for assessing strength of brittle materials is the bending (flexure) test, as it involves simple sample shapes. The loading configuration is either three- or four-point bending. The bend strength is determined from the maximum applied tensile stress, assuming that the material fails in tension. Typically brittle materials contain flaws varying in size and type, which causes strength to vary from sample to sample. This variability in strength is usually expressed in terms of a failure probability. Since the form of the strength distribution is not known a priori, an empirical distribution, first suggested by Weibull [1] is often used. Once the strength of a given material is fitted to this distribution, the failure probability can be predicted and, in case of being too high, it will clearly impact safety, so either the design needs to be changed or the material improved. The Weibull approach assumes a simple power-law stress function for the survival of parts, which is integrated over the sample volume. As such, the three-parameter Weibull distribution for a body failing under a tensile stress $\sigma$ can be written as:

$$F = 1 - \exp \left[ - \int_{\sigma_{\min}}^{\sigma} \left( \frac{\sigma - \sigma_{\min}}{\sigma_{0}} \right)^m d\sigma \right]$$

(1)

where $F$ is the failure probability, $m$ the Weibull modulus, $\sigma_0$ the characteristic strength and $\sigma_{\min}$ the minimum strength. The Weibull modulus describes the width of the strength distribution; the higher the value of $m$, the lower is the strength variability. The purpose of this study is to help improve the knowledge of the mechanical properties of a mortar.
based on Portland cement by studying its three-point bending strength behaviour, as well as its elastic and fracture toughness properties. Some examples where flexural strength data are of crucial importance in construction include road pavements and airstrips. This study has some significance because flexural strength analysis using Weibull statistics applied to mortars from tests conducted under standard conditions, as described in this paper, is sparse.

2. Experimental

To prepare each group of 3 samples (Fig. 1), according to the EN196-1 standard [2], from a total of 24 samples (40x40x160 mm), 1350 g of sand, 450 g of cement and 225 g of deionized water were used. The sand had a maximum particle size of 2.36 mm and the Portland cement was CEM I 42.5R.

Flexural strength measurements were carried out in an Ibertest testing machine (Madrid, Spain), model Autotest 300, with a maximum capacity in flexure of 15 kN, located at SECIL, after 28 days of curing time, in three-point bending (Fig. 2), according to the above mentioned standard [2]:

$$\sigma_0 = \frac{1.5FL}{bh^2}$$

(2)

where $\sigma_0$ is the flexural strength (MPa), F the maximum load (N), b the width (mm), h the height (mm) and L the outer span (100 mm).

All tests were performed at a loading rate of 50±10 N/s. The maximum stress, in this type of loading geometry, is present only at the centre of the beam. After testing, however, it was noted that some samples did not break at maximum load (i.e., L/2). Hence, the $\sigma_0$ values were corrected by measuring the actual position where fracture was initiated and so the true strengths are lower.

Both set of data obtained according to the standard ($\sigma_0$) and after correction ($\sigma_0^*$) were treated using the two-parameter Weibull statistics [1], which is assumed in many cases, where $\sigma_{min}=0$.

The most widely used method to estimate the two parameters from a set of experimentally measured fracture stresses is by using linear regression, involving the ranking of the strength data in an ascending order and the assignment of a probability of failure to each sample. If that is the case, to analyse strength data, Eq. (1) can be simplified as follows:

$$y = \ln \ln \frac{1}{P_s} = m \ln \sigma - m \ln \sigma_0$$

(3)

which represents a linear relationship between y and $\ln\sigma$, with slope m. In such a procedure, a failure probability is needed for each test sample. Thus, the experimental survival probability ($P_s$) is usually estimated as [3]:

$$P_s = 1 - \left(1 - \frac{0.5}{n}\right)$$

(4)

where i is the rank of $\sigma$-value when all flexural strength results (of the same group of samples) are positioned in increasing order (from weakest to strongest), and n the total number of results. This estimator has been shown to give the least biased estimation of m when n>20 [3]. In this case, the so-called Weibull modulus (m) is a measure of scatter and is roughly inversely related to the coefficient of variation, $C_v$:

$$m \approx \frac{1.2}{C_v}$$

(5)

with

$$C_v = \frac{s}{\bar{\sigma}}$$

where $\bar{\sigma}$ is the mean strength and s the standard deviation.
The dynamic Young’s modulus was measured at room temperature by the impulse of excitation method, according to ASTM C1259-96 [4], using an acoustic device developed by IMCE nv – Integrated Material Control Engineering (Diepenbeek, Belgium) located at LNEG. The Young’s modulus is determined using the resonant frequency in the flexural mode of vibration according to the following equation:

\[ E = 0.9465 \left( \frac{m f_r^2}{b} \right) \left( \frac{L^3}{t^3} \right) T_1 \]  

(6)

where \( E \) is the Young’s modulus (Pa), \( m \) the mass of the bar (g), \( b \) the width of the bar (mm), \( L \) the length of the bar (mm), \( t \) its thickness (mm), \( f_r \) the fundamental resonant frequency of the bar in flexure (Hz), and \( T_1 \) a correction factor for fundamental flexural mode of vibration which is calculated as follows:

\[ T_1 = 1 + 6.585 \left( 1 + 0.0752v + 0.8109v^2 \right) \left( \frac{t}{L} \right)^2 - 0.868 \left( \frac{t}{L} \right)^4 - \frac{8.340(1 + 0.2023v + 2.173v^2)}{1.000 + 6.338(1 + 0.1408v + 1.536v^2)} \] 

(7)

with \( v \) being the Poisson’s ratio. Assuming that the material is isotropic, \( v=0.30 \). The \( f_r \) values were determined by placing the samples on supports located at the fundamental nodal points (0.224 \( L \) from each sample end). For each sample, a minimum of three measurements of \( f_r \) were performed.

Fracture toughness was determined, based on critical crack sizes measured by fractography, in fracture surfaces of samples according to the ASTM C 1322-05b standard [5] using Griffith approach:

\[ K_{ic} = Y\sigma_\varepsilon \sqrt{a} \]  

(8)

where \( Y \) is a dimensionless value that depends on the crack shape and \( a \) is the crack depth.

### 3. Results and Discussion

Flexural strength data prior and after correction are listed in Table 1. A typical broken sample is shown in Fig. 3. Based on Eq. (5), values of \( m \) were estimated as 28 (when strength values were determined according to EN196-1 Standard) and 22 (for the set of corrected values).

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These values of \( m \) are in excellent agreement with those predicted by Eq. (3). Indeed, a Weibull plot (Fig. 4), showing the two sets of flexural data obtained according to the EN196-1 standard and corrected values taking into account the actual position where failure origin took place, can be fitted applying a linear regression, which gives an estimate of Weibull modulus ranging from 28 to 22, respectively. In accordance to Eq. (5), since \( s \) increases due to the fact that not all the samples did
break at maximum stress, then \( m \) decreases. For metals and alloys, \( m \approx 100 \). For traditional ceramics (e.g., bricks, pottery, glasses), \( m \) is lower than 3. Engineering ceramics, in which the process is better controlled (so that the number of flaws is expected to be lower), have \( m \) values in the range of 5-20. Hence, the estimated \( m \) values are very high when compared to those of typical ceramic materials, suggesting that the mortar under study has a narrow flaw size distribution.

From the intercept \( \ln \sigma_0 = -46.14 \), one obtains a specific characteristic strength \( \sigma_0^* = 8.15 \) MPa, which corresponds to the value of \( \sigma_0^* \) that causes the failure of \( 1/e = 0.632 \), i.e., 63.2% of samples.

One of the basic parameters in the resistance of the building materials is the Young’s modulus, which indicates the deformation capability of a given material in its linear elastic regime depending on the strain to which it is subjected. The dynamic Young’s modulus of the mortar under investigation was determined to be 34.1±0.1 GPa. This value is higher than those found in the literature for air lime-Portland cement mortars (9-13 GPa, depending on their composition) measured with the same technique [5]. Elastic modulus derived from compression stress-strain curves for cement mortars with the same water/cement ratio and sand/cement ratio of 2.28 (3 in this study) were found to range between 24.0 and 37.6 GPa [6], suggesting that the measured \( E \) values lie within the expected range.

Judging from the calculated standard deviation (0.1 GPa), it may be concluded that the mortar under investigation is quite homogenous.

For the uniaxial stress geometry considered, fracture occurs under pure mode I. Thus, the failure criterion can be described in accordance with Eq. (8).

Based on the observed configurations of the initial cracks in sample, a compliance value of \( Y = 1.59 \) was assumed in accordance with the ASTM C1322-05b standard [7].

Taking into account the crack depth \( (a) \), with values (0.7-1.1 mm) measured by fractography analysis, the mean \( K_{IC} \) value was determined to be 0.37±0.04 MPa m\(^{1/2}\). Fig. 5 presents a typical example of a fracture surface showing that the critical defect size is around 1 mm (dashed semi-ellipse indicates the surface crack position).

The fracture toughness values obtained are similar to those measured for another mortar \( (0.30±0.08 \text{ MPa m}^{1/2}) \) using the single-edge notch beam method [8].

To the authors’ knowledge, only a few works related to \( K_{IC} \) measurements applied to mortars can be found in the literature using both a notched beam and a double cantilever beam method [9,10]. Similar \( K_{IC} \) values to those obtained in this study were reported. This is not surprising since mortars are not usually expected to be used under uniaxial tensile mode. Such low \( K_{IC} \) values are typical of silicate glasses (0.6-0.8 MPa m\(^{1/2}\)).

4. Conclusions

From this study, the following conclusions could be drawn:

- The mean corrected flexural strength is lower than that determined by the EN196-1 standard;
- The scatter of the flexural results is higher for the corrected data set and consequently the Weibull modulus (22) is lower than that obtained for the set of data obtained according to the above mentioned standard (28);
- Judging from Weibull modulus and Young’s modulus data scatter, it can be concluded that the...
developed mortar is quite homogenous;

- The mean fracture toughness of the mortar is 0.37±0.04 MPa m$^{1/2}$, which is typical of very brittle materials, such as silicate glasses.

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References